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UNITARY SYMMETRY FOR PEDESTRIANS

by

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UNITARY SYMMETRY FOR PEDESTRIANS
(or, I-SPIN, U-SPIN, V-ALL SPIN FOR I-SPIN)

by

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ABSTRACT

Unitary symmetry of elementary particles is described in a simple way requiring no knowledge of group theory. The basic approach is that of Levinson, Lipkin and Meshkov, and uses SU_2 subgroups of the group SU_3. Unitary symmetry appears as a generalization of I-spin symmetry, in which three "spins" are defined in a symmetrical way and are all required simultaneously to be conserved. One of the three spins is I-spin; the other two, called U spin and V spin, define certain transformations between particles of different strangeness. Experimental consequences of the conservation of all three spins are calculated with conventional angular momentum algebra. The breaking of unitary symmetry by electromagnetic interactions and by the strong interactions producing mass splittings leads to consequences which are easily calculated with the use of the three spins and angular momentum algebra.

UNITARY SYMMETRY FOR PEDESTRIANS*

(or, I-SPIN, U-SPIN, V-ALL SPIN FOR I-SPIN)

H. J. Lipkin

INTRODUCTION

Unitary symmetry theories describe elementary particles in terms of a higher symmetry including I-spin symmetry. The essentail features of this higher symmetry are manifest in Figs. 1(a, b, and c). On diagrams of hypercharge Y versus the z-component of the I-spin, T_z, points are plotted respectively for (a) all known baryons of spin-1/2, (b) all known pseudoscalar mesons, and (c) all known vector meson resonances. These plots are all seen to have a 120° rotation symmetry.

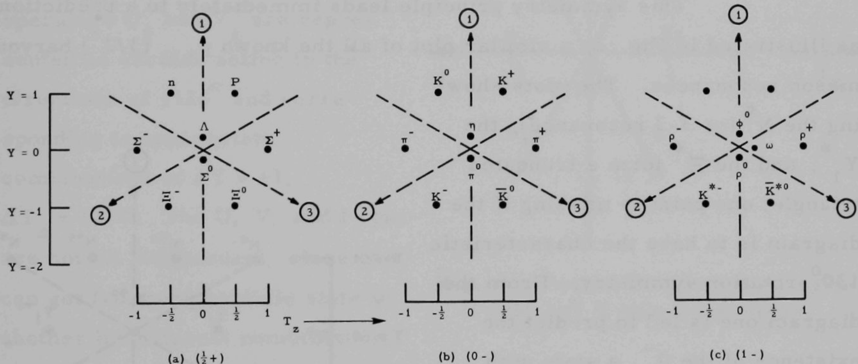

Fig. 1. Y vs. T_z plots for stable baryons and pseudoscalar and vector mesons.

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* This was originally a seminar lecture presented at Argonne in August 1963.

Noting that particles appearing along the same horizontal line (same Y, different T_z) constitute I-spin multiplets, one can describe the symmetry as follows. The horizontal direction and directions along lines at $\pm 120^{\circ}$ seem to be equivalent in some manner. Thus particles might be grouped into multiplets along lines at $\pm 120^{\circ}$ as well as horizontally. There would then be three kinds of multiplets, I-spin and two others which could be called U-spin and V-spin.

Let us now investigate the consequences of extending I-spin symmetry to include U spin and V spin in a symmetrical way. We should first expect that all strongly interacting particles and resonances should be classified into supermultiplets or unitary multiplets like those exhibited in Figs. 1(a, b, and c), all having the same 120°-rotation symmetry on a plot of Y versus T_z. Such unitary multiplets would contain several I-spin multiplets and exactly the same number and kind of U-spin and V-spin multiplets.

This symmetry principle leads immediately to a prediction as illustrated in Fig. 2, a similar plot of all the known $p_{3/2}$ $(3/2^{+})$ baryon-meson resonances. The plots showing the N^* (or 3-3 resonance), the Y_1^*, and the Ξ^* form a truncated triangle; one point is missing if the diagram is to have the characteristic 120°-rotation symmetry. From the diagram one is led to predict the existence of the Ω^-, a state with spin 3/2, even parity, hypercharge Y = -2 (or strangeness S = -3), negative charge, and I spin 0.

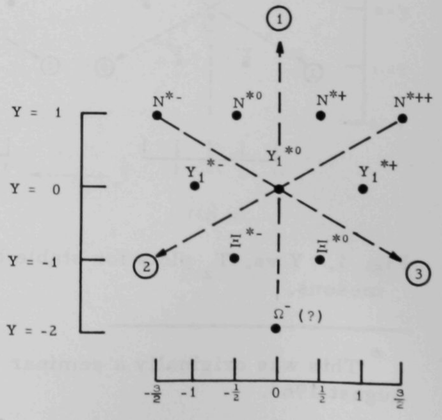

Fig. 2. Y vs. T_z plot for (3/2+) baryon resonances.

Let us now attempt to describe this symmetry in more detail so that we can look for further predictions in addition to the existence of particles in unitary multiplets.

In analogy to the example of I spin, we can construct a formalism in which U spin and V spin are defined and are conserved in the same way as I-spin. Let us define U-spin and V-spin operators which satisfy commutation rules, such as those for angular momenta, in the same manner as the I-spin operators. This is represented diagramatically in Fig. 3. The I-spin operators τ_+ and τ_- do not change Y, but are step operators on T_z, changing the eigenvalue by ± 1. These are represented by vectors corresponding to appropriate combinations of $\Delta Y = 0$, $\Delta T_z = \pm 1$. The U-spin and V-spin step operators $U_\pm$ and $V_\pm$ are represented as vectors acting in the directions of $\pm 120^o$ and corresponding to appropriate combinations of $\Delta Y = \pm 1$, $\Delta T_z = \pm 1/2$. The U, V, and I spin are not all independent, since one can get from one particle state to another by different combinations of these operators. The I-spin, U-spin, and V-spin operators thus do not necessarily commute with one another; consistent commutation relations must be found. One way to find them is by use of group theory. The algebra of the set of six step operators plus two diagonal operators Y and T_z is known to group theorists as the algebra of the group SU_3. However, many experimental consequences of unitary symmetry can be obtained without use of SU_3 group theory by classifying particles into multiplets

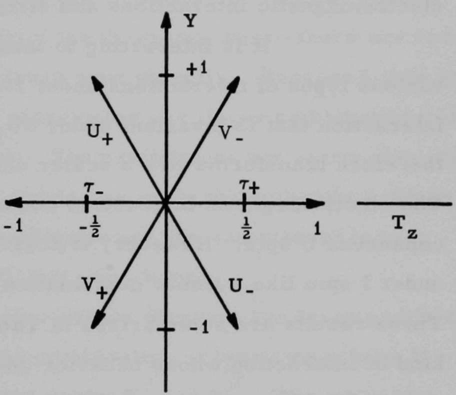

Fig. 3. Vector Diagram for I spin, U spin and V spin Generators.

and requiring conservation of U spin and V spin as well as I spin, i.e., by dealing only with the conventional algebra of angular momenta.

The electromagnetic properties of U spin are of particular interest. The U-spin operators U_+ and U_- change the strangeness of a particle, but not its electric charge. U-spin multiplets thus consist of particles all having the same coupling to the electromagnetic field. The electromagnetic interaction therefore conserves U spin rigorously and the photon can be considered to be a particle of U spin zero. The addition of strong interactions invariant under SU_3 (and therefore conserving I spin and V spin as well as U spin) does not affect U-spin conservation in electromagnetic interactions. Thus U spin is conserved in any combination of electromagnetic interactions and strong interactions invariant under SU_3.

It is interesting to tabulate the transformation properties of various types of interactions under I-spin and U-spin transformations. An interaction that is invariant under SU_3 conserves both I spin and U spin and therefore transforms like a scalar under both I-spin and U-spin transformations. The electromagnetic interaction transforms like a scalar under U spin and conserves U spin. However, it does not conserve I spin and transforms under I spin like a linear combination of an isoscalar and an isovector. These results are summarized in Table I. One might imagine a different kind of interaction whose behavior under I-spin and U-spin transformations would be the converse of the electromagnetic interaction; i.e., it would conserve I spin but not U spin and would transform like a linear combination of a U-spin scalar and a U-spin vector. Such an interaction is denoted by M in Table I. We shall see below that this kind of interaction is of interest in the study of strongly interacting particles.

TABLE I. Transformation Properties of Several Interactions
S = Scalar; V = Vector

Interaction	I-Spin	U-Spin
Invariant under SU_3	S (conserved)	S (conserved)
Electromagnetic	S + V	S (conserved)
M	S (conserved)	S + V

Figures 4(a, b, c, and d) in the Appendix illustrate classification of particles in U-spin multiplets and are obtained from the corresponding diagrams of Fig. 1 by a 120° rotation — with one small difference. The points in the center of the diagrams where there are two particles such as Λ and Σ^0 are not taken over directly. Because I spin and U spin do not commute, the U-spin eigenstates are linear combinations of the corresponding I-spin eigenstates. The coefficients are easily obtained with simple algebra.* At all other points, at which there is only a single particle at each point, there is no ambiguity and the same particle is a simultaneous eigenstate of I spin, U spin, and V spin.

The center of the vector-meson diagram has three particles, the ρ^0, the ω, and the ϕ. One linear combination of these must have U = 1, and must belong to the same multiplet as the K^0 and $\overline{K^0}$. The other two must have U = 0. SU_3 algebra requires that one linear combination of the ω and ϕ should be a U = 0, T = 0 unitary singlet which should be separated from the remaining eight states. This particular linear combination is not determined by SU_3 algebra and has been denoted by Sakurai as

$$|T = 0,\ U = 0\rangle = \cos\lambda\,|\omega\rangle + \sin\lambda\,|\phi\rangle \equiv \omega^{(0)}.$$

---

* This point is discussed in the Appendix.

Here λ is a parameter determined from experimental data (such as mass splittings) to be about 38°. The corresponding orthogonal linear combination

$$|T = 0, \text{ octet}\rangle = -\sin\lambda\,|\,\omega\,\rangle + \cos\lambda\,|\phi\,\rangle \equiv \phi^{(0)}$$

then belongs in an octet of vector mesons similar to the octet of baryons and pseudoscalar mesons. This linear combination of ω and ϕ then combines with the ρ^0 to from U-spin eigenstates in the same way as the Λ combines with the Σ^0.

Note that in Figs. 4 the vertical co-ordinate turns out to be just the electric charge Q, analogous to the hypercharge Y in Figs. 1. The z component of U spin is a linear combination of Y and Q, namely

$$U_z = Y - \tfrac{1}{2}Q.$$

Component U_z has no particular physical significance, other than obviously being conserved in any process in which Y and Q are conserved.

From the U-spin classification of particles, the consequences of U-spin conservation are easily obtained in the same manner as the consequences of I-spin conservation. This is illustrated by the following examples.

1. PRODUCTION OF BARYON RESONANCES IN MESON-BARYON REACTIONS

Consider the four reactions:

$$\pi^- + p \to K^+ + Y_1^{*-} ,$$

$$\pi^- + p \to \pi^+ + N^{*-} ,$$

$$K^- + p \to K^+ + \Xi^{*-} ,$$

$$K^- + p \to \pi^+ + Y_1^{*-} .$$

$$U: \quad \underbrace{1/2 \quad 1/2}_{0 \text{ or } 1} \quad \underbrace{1/2 \quad 3/2}_{1 \text{ or } 2}$$

$$U_{total}:$$

The π^- and K^- belong to the same U-spin doublet with U = 1/2. The proton is a member of a U-spin doublet with U = 1/2. There are thus two possible U-spin states for the left-hand side of these reactions, namely U = 0 and U = 1. The K^+ and π^+ are members of the same U-spin doublet with U = 1/2. The Y^{*-}, N^{*-}, and Ξ^{*-} are all members of a U-spin quartet with U = 3/2. The possible U-spin states obtained from coupling U = 1/2 with U = 3/2 are U = 1 and U = 2. If U-spin is conserved in these reactions, there is thus only one possible U-spin channel that is common to both sides of the reactions, namely U = 1. The amplitudes for these four reactions are therefore all expressed in terms of a single parameter, the amplitude for the U = 1 channel. The coefficients are products of two Clebsch-Gordan coefficients, one for each side of a reaction. The coefficient for the left-hand side describes the coupling of two spins of 1/2 to a total spin 1; the one for the right-hand side describes the coupling of a spin of 1/2 and a spin of 3/2 to a total spin 1. These amplitudes are

$$
\begin{aligned}
\langle \pi^- p | K^+ Y_1^{*-} \rangle &= \left(\tfrac{1}{2}\ \tfrac{1}{2}\ \ \tfrac{1}{2}\ \tfrac{1}{2} \middle| 1\ 1\right)\left(\tfrac{1}{2}\ \tfrac{3}{2}\ \ \tfrac{1}{2}\ \tfrac{1}{2} \middle| 1\ 1\right) a_1 = -\tfrac{1}{2}\, a_1 \\
\langle \pi^- p | \pi^+ N^{*-} \rangle &= \left(\tfrac{1}{2}\ \tfrac{1}{2}\ \ \tfrac{1}{2}\ \tfrac{1}{2} \middle| 1\ 1\right)\left(\tfrac{1}{2}\ \tfrac{3}{2}\ -\tfrac{1}{2}\ \tfrac{3}{2} \middle| 1\ 1\right) a_1 = \tfrac{\sqrt{3}}{2}\, a_1 \\
\langle K^- p | K^+ \Xi^{*-} \rangle &= \left(\tfrac{1}{2}\ \tfrac{1}{2}\ -\tfrac{1}{2}\ \tfrac{1}{2} \middle| 1\ 0\right)\left(\tfrac{1}{2}\ \tfrac{3}{2}\ \ \tfrac{1}{2}\ -\tfrac{1}{2} \middle| 1\ 0\right) a_1 = -\tfrac{1}{2}\, a_1 \\
\langle K^- p | \pi^+ Y^{*-} \rangle &= \left(\tfrac{1}{2}\ \tfrac{1}{2}\ -\tfrac{1}{2}\ \tfrac{1}{2} \middle| 1\ 0\right)\left(\tfrac{1}{2}\ \tfrac{3}{2}\ -\tfrac{1}{2}\ \tfrac{1}{2} \middle| 1\ 0\right) a_1 = \tfrac{1}{2}\, a_1 \,.
\end{aligned}
\tag{1.1}
$$

2. PHOTOPRODUCTION OF N^* AND Y^*

Consider the two reactions:

$$
\begin{aligned}
&\gamma + p \to N^{*0} + \pi^+ , \\
&\gamma + p \to Y^{*0} + K^+ .
\end{aligned}
$$

$$
\begin{array}{lcc}
U: & \underbrace{0 \quad 1/2}_{1/2} & \underbrace{1 \quad\quad 1/2}_{1/2 \text{ or } 3/2} \\
U_{total}: & &
\end{array}
$$

The (γ, p) system is a U-spin eigenstate with $U = 1/2$ since the photon has $U = 0$ and the proton has $U = 1/2$. The N^{*0} and the Y_1^{*0} are members of a $U = 1$ triplet. The π^+ and K^+ are members of a $U = 1/2$ doublet. If U spin is conserved in these reactions, there is only one possible U-spin channel, namely $U = 1/2$. The branching ratio for the two reactions thus depends only on the Clebsch-Gordan coefficients describing the coupling of a spin 1 and a spin 1/2 to a total spin 1/2. It is

$$
\frac{\langle \gamma p | N^{*0}\, \pi^+ \rangle}{\langle \gamma p | Y^{*0}\, K^+ \rangle} = \frac{(1\ \tfrac{1}{2}\ \ 1\ -\tfrac{1}{2} \,|\, \tfrac{1}{2}\ \tfrac{1}{2})}{(1\ \tfrac{1}{2}\ \ 0\ \ \tfrac{1}{2} \,|\, \tfrac{1}{2}\ \tfrac{1}{2})} = -\sqrt{2}\,. \tag{2.1}
$$

3. MESON PHOTOPRODUCTION

Consider the reactions

$$\gamma + p \rightarrow n + \pi^+ ,$$

$$\gamma + p \rightarrow \Lambda + K^+ ,$$

$$\gamma + p \rightarrow \Sigma^0 + K^+ .$$

$$U: \quad \underbrace{0 \quad 1/2}_{1/2} \qquad \underbrace{1 \text{ or } 0 \quad 1/2}_{1/2 \text{ or } 3/2}$$

$$U_{total}:$$

Here again, the left-hand side has U = 1/2 and is a pure U-spin eigenstate. The situation on the right-hand side of these reactions is more complicated because the Λ and Σ^0 are not U-spin eigenstates but are mixtures of U = 0 and U = 1. These particles can combine in two ways to make a U = 1/2 state. Either the U = 0 or the U = 1 component of the Λ and Σ^0 can be coupled to the U = 1/2 meson to obtain a total U spin of 1/2. There are therefore two independent complex amplitudes describing these three reactions. The existence of two complex amplitudes implies three real parameters: two magnitudes and one relative phase. Since there are only three cross sections, one cannot relate these <u>cross sections</u> by an equality. However, the relation between the <u>amplitudes</u> for the three reactions can be obtained and leads to inequalities relating the cross sections. These relations are obtained most easily by noting that the linear combination

$$\tfrac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda$$

is a U-spin eigenstate and belongs to the same U = 1 triplet as the neutron. The amplitudes for the photoproduction of this particular linear combination and for neutron production thus are related by the

Clebsch-Gordan coefficient describing the coupling of a spin 1 with a spin 1/2 to a total spin 1/2. The ratio of amplitudes therefore is

$$\frac{\langle \gamma\, p | n\, \pi^+ \rangle}{\langle \gamma\, p | \{\tfrac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda\} K^+ \rangle} = -\sqrt{2} \tag{3.1}$$

so that

$$\langle \gamma\, p | n\, \pi^+ \rangle = -\frac{\sqrt{2}}{2} \langle \gamma\, p | \Sigma^0 K^+ \rangle - \frac{\sqrt{6}}{2} \langle \gamma\, p \, | \Lambda K^+ \rangle . \tag{3.2}$$

This relation between <u>amplitudes</u> is not a relation between <u>cross sections</u> because the relative phase of the Σ^0 and Λ production amplitudes is unknown. However, it follows that the absolute values of the transition-matrix elements are related by the inequalities

$$| \langle \gamma p | n\, \pi^+ \rangle | \leq \frac{\sqrt{2}}{2} |\langle \gamma p | \Sigma^0 K^+ \rangle | + \frac{\sqrt{6}}{2} | \langle \gamma p | \Lambda K^+ \rangle | , \tag{3.3a}$$

$$| \langle \gamma p | n\, \pi^+ \rangle | \geq \left| \frac{\sqrt{2}}{2} |\langle \gamma p | \Sigma^0 K^+ | - \frac{\sqrt{6}}{2} | \langle \gamma p | \Lambda K^+ \rangle | \right| . \tag{3.3b}$$

4. ELECTROMAGNETIC DECAYS OF THE N^*, Y^*, AND Ξ^*

The Y_1^{*-} and Ξ^{*-} belong to a U = 3/2 quartet, while the Y_1^{*+} and N^{*+} belong to a U = 1/2 doublet. The Σ^-, Ξ^-, Σ^+, and p all have U = 1/2. Thus we find that the electromagnetic decays of the Y^{*-} and Ξ^{*-} are forbidden but the decays of the Y^{*+} and N^{*+} are allowed and have equal amplitudes, i.e.,

<u>Forbidden</u>: $\langle Y_1^{*-} | \Sigma^- \gamma \rangle$ and $\langle \Xi^{*-} | \Xi^- \gamma \rangle$

<u>Allowed</u>: $\langle Y_1^{*+} | \Sigma^+ \gamma \rangle = \langle N^{*+} | p \gamma \rangle .$ (4.1)

The N^{*0}, Ξ^{*0}, and Y^{*0} belong to the same U = 1 triplet and can decay electromagnetically to the corresponding members of the U = 1 baryon triplet consisting of the n, the Ξ^0, and the linear combination $\frac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda$. Thus

$$\langle N^{*0} | n\gamma\rangle = \langle \Xi^{*0} | \Xi^0 \gamma\rangle = 2\langle Y^{*0} | \Sigma^0 \gamma\rangle = \frac{2}{\sqrt{3}} \langle Y^{*0} | \Lambda\gamma\rangle . \quad (4.2)$$

5. ELECTROMAGNETIC DECAYS OF THE η AND π^0

Since photons have U = 0, only a U = 0 state can decay into two photons. The η and π^0 are both mixtures of U = 0 and U = 1. The U = 0 state is the linear combination

$$\frac{\sqrt{3}}{2} | \pi^0 \rangle - \frac{1}{2} | \eta \rangle .$$

Thus,

$$\langle \pi^0 | 2\gamma \rangle = - \sqrt{3} \langle \eta | 2\gamma \rangle . \quad (5.1)$$

6. MULTIPARTICLE REACTIONS; TWO-MESON PHOTOPRODUCTION

U-spin conservation is particularly useful in considering reactions in which several particles are produced in the final state. If SU_3 algebra is used and all possible final states are considered, there are many possible couplings and many channels. Most relations obtained in this way are complicated. U spin provides a method of choosing the particular reactions having simple properties; i.e., those reactions for which only one or two couplings are allowed by U-spin conservation. Consider for example the reactions

$$\gamma + p \to N^{*-} + \pi^+ + \pi^+ ,$$

$$\gamma + p \to Y^{*-} + K^+ + \pi^+ ,$$

$$\gamma + p \to \Xi^{*-} + K^+ + K^+ .$$

$$U: \quad 0 \quad 1/2 \quad 3/2 \quad 1/2 \quad 1/2$$

U-spin conservation requires that the two mesons should couple to U = 1 in order that the combination may couple with the U = 3/2 baryon resonances to a total U = 1/2. Thus

$$\langle \gamma p \,|\, N^{*-} \pi^+ \pi^+ \rangle / \langle \gamma p \,|\, Y^{*-} \frac{1}{\sqrt{2}} \{ K^+ \pi^+ + \pi^+ K^+ \} \rangle / \langle \gamma p \,|\, \Xi^{*-} K^+ K^+ \rangle$$

$$= \left(\frac{3}{2} \;\; 1 \;\; \frac{3}{2} \;\; -1 \,\middle|\, \frac{1}{2} \;\; \frac{1}{2} \right) \Big/ \left(\frac{3}{2} \;\; 1 \;\; \frac{1}{2} \;\; 0 \,\middle|\, \frac{1}{2} \;\; \frac{1}{2} \right) \Big/ \left(\frac{3}{2} \;\; 1 \;\; -\frac{1}{2} \;\; 1 \,\middle|\, \frac{1}{2} \;\; \frac{1}{2} \right) \tag{6. 1}$$

$$= \quad 1/\sqrt{2} \quad / \quad -1/\sqrt{3} \quad / \quad 1/\sqrt{6} \; .$$

The K^+ and π^+ are spinless bosons belonging to the same U-spin doublet. The wave function for a two-particle $K^+\pi^+$ system must be totally symmetric in space and U spin. Since the U = 1 state, $(1/\sqrt{2}) \; \{K^+\pi^+ + \pi^+K^+\}$ is symmetric in U spin, it must alse be space-symmetric. The angular distribution for this reaction is therefore symmetric under interchange of the two mesons. They must be in a state of even orbital angular momentum and even parity with respect to their center of mass.

Similar relations follow for the production of the corresponding vector mesons.

7. PRODUCTION OF Ω^- AND BARYON RESONANCES IN PROTON-ANTIPROTON ANNIHILATION

The Ω^-, N^{*-}, Y^{*-}, and Ξ^{*-} are seen from Fig. 2 to be in the same U = 3/2 quartet. The corresponding antibaryons also form a U-spin quartet. Consider the reactions

$$\begin{aligned}
\bar{p} + p &\rightarrow N^{*-} + \overline{N^{*-}}, \\
\bar{p} + p &\rightarrow Y^{*-} + \overline{Y^{*-}}, \\
\bar{p} + p &\rightarrow \Xi^{*-} + \overline{\Xi^{*-}}, \\
\bar{p} + p &\rightarrow \Omega^- + \overline{\Omega^-} .
\end{aligned}$$

$$U: \quad \underbrace{1/2 \quad 1/2}_{0 \text{ or } 1} \qquad \underbrace{3/2 \qquad 3/2}_{0,\ 1,\ 2 \text{ or } 3} \qquad (U_{total})$$

The proton and antiproton are both in U-spin doublets having U = 1/2. The reactions above therefore go through two U-spin channels (U = 0 and U = 1) and are expressed in terms of two independent complex amplitudes. Letting a_0 and a_1 be the amplitudes for the U = 0 and U = 1 channels, we can write

$$\begin{aligned}
(\bar{p}\, p \,|\, N^{*-}\ \overline{N^{*-}}) &= -\frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,\frac{3}{2}\,-\frac{3}{2}\middle|\,0\ 0\right)a_0 + \frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,\frac{3}{2}\,-\frac{3}{2}\middle|\,1\ 0\right)a_1, \\
(\bar{p}\, p \,|\, Y^{*-}\ \overline{Y^{*-}}) &= -\frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\middle|\,0\ 0\right)a_0 + \frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,\frac{1}{2}\,-\frac{1}{2}\middle|\,1\ 0\right)a_1, \\
(\bar{p}\, p \,|\, \Xi^{*-}\ \overline{\Xi^{*-}}) &= -\frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,-\frac{1}{2}\,\frac{1}{2}\middle|\,0\ 0\right)a_0 + \frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,-\frac{1}{2}\,\frac{1}{2}\middle|\,1\ 0\right)a_1, \\
(\bar{p}\, p \,|\, \Omega^-\ \overline{\Omega^-}) &= -\frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,-\frac{3}{2}\,\frac{3}{2}\middle|\,0\ 0\right)a_0 + \frac{1}{\sqrt{2}}\left(\frac{3}{2}\,\frac{3}{2}\,-\frac{3}{2}\,\frac{3}{2}\middle|\,1\ 0\right)a_1,
\end{aligned} \tag{7. 1}$$

where the factors $\pm(1/\sqrt{2})$ come from coupling $(\bar{p}\,p)$ to $U = 1$ and $U = 0$.

When the numerical values for the Clebsch-Gordan coefficients are inserted, these become

$$\begin{aligned}
\sqrt{2}(\bar{p}\,p\,|\,N^{*-}\,\overline{N^{*-}}) &= -\tfrac{1}{2}a_0 + \frac{3}{2\sqrt{5}}a_1 , \\
\sqrt{2}(\bar{p}\,p\,|\,Y^{*-}\,\overline{Y^{*-}}) &= +\tfrac{1}{2}a_0 - \frac{1}{2\sqrt{5}}a_1 , \\
\sqrt{2}(\bar{p}\,p\,|\,\Xi^{*-}\,\overline{\Xi^{*-}}) &= -\tfrac{1}{2}a_0 - \frac{1}{2\sqrt{5}}a_1 , \\
\sqrt{2}(\bar{p}\,p\,|\,\Omega^{-}\,\overline{\Omega^{-}}) &= +\tfrac{1}{2}a_0 + \frac{3}{2\sqrt{5}}a_1 .
\end{aligned} \tag{7.2}$$

These four relations involve three parameters: the magnitudes of the amplitudes a_0 and a_1 and the relative phase. Eliminating these three parameters between the four equations gives a relation between the cross sections which can be written as an expression for the $(\Omega^-, \overline{\Omega^-})$ production cross sections expressed in terms of the other three, namely

$$\sigma(\Omega^-\,\overline{\Omega^-}) = \sigma(N^{*-}\,\overline{N^{*-}}) + 3\,\{\sigma(\Xi^{*-}\,\overline{\Xi^{*-}}) - \sigma(Y^{*-}\,\overline{Y^{*-}})\} . \tag{7.3}$$

8. PESHKIN'S THEOREM AND GENERAL RELATIONS FOR Ω^- PRODUCTION

The relation (7.3) between the cross sections for production of Ω^-, N^{*-}, Y^{*-}, and Ξ^{*-} can be generalized by the use of Peshkin's symmetry theorem[1] to obtain

$$\sigma(\Omega^-) = \sigma(N^{*-}) + 3\{\sigma(\Xi^{*-}) - \sigma(Y^{*-})\} . \tag{8.1}$$

---

[1] M. Peshkin, Phys. Rev. 121, 636 (1961), Eq. (4.4).

This relation is valid for $(\bar{p}p)$ and (K^-p) reactions, for (γp) and (γn) photoproduction reactions, and also in (π^-p), (π^+p), and (K^+p) reactions. The validity of the result depends only upon conservation of U spin. It applies to total cross sections for the production of these particles along with any other particles, and also to partial cross sections provided that the set of partial cross sections chosen is consistent with U-spin symmetry; i.e., provided that they include all possible reactions involving members of a given set of U-spin multiplets.

Equation (8.1) is obtained directly from Peshkin's formula by substituting U spin instead of I spin. The relation is directly analogous to the well-known result in angular momentum that a particle of spin 1 can have no moment higher than a quadrupole moment and in particular that its octupole moment must be zero. In all of the reactions considered, the initial states are either of $U = 1/2$ or mixtures of $U = 0$ and $U = 1$. There is never any component having $U > 1$. Since U spin is conserved in the transition, it follows that the expectation of any "U-spin tensor" of rank greater than 2 must vanish in the final state. In particular, the expectation value of the "U-spin octupole" tensor of rank 3 must vanish. Equation (8.1) expresses the fact that the particular linear combination of the probabilities of finding a Ω^-, an N^{*-}, a Ξ^{*-}, and a Y^{*-} transforms like a third-rank U-spin tensor and therefore has zero expectation value in the final state.

The breaking of SU_3 symmetry and the consequent mass splittings must be considered in the use of this formula. The various reactions that give rise to the cross sections must be corrected for mass-difference effects such as differences in available phase space. Of particular importance are cases in which certain channels are closed because of mass-difference effects while others in the same U-spin multiplet are open.

The above restrictions apply only to the closing of channels by mass and energy effects that violate U-spin conservation. If a channel is closed because of conservation of strangeness, which is consistent with

U-spin conservation, the validity of the relation is unaffected. For example, the N^{*-}, the Ξ^{*-}, and the Y^{*-} can be produced in $(\pi^- p)$ reactions along with two mesons, but the Ω^- cannot because it requires at least three K mesons in order to conserve strangeness. The relation (8.1) still holds for this case. Since $\sigma(\Omega^-)$ is zero, the result gives a relation between the cross sections for the three other processes.

The relation (8.1) can thus be applied to the $(\pi^- p)$ reactions, in which the Ω^- is <u>not</u> produced, in order to check the effect of the symmetry-breaking terms and to see whether or not these can be treated in a consistent fashion. The results can then be applied to estimate $\sigma(\Omega^-)$ in the $(K^- p)$ system, corrections for mass splittings being applied in analogy to those found to hold in the $(\pi^- p)$ system.

9. THREE-MESON COUPLINGS

Three-meson vertices are of interest both in the consideration of the decay of one meson into two other mesons and also as parts of diagrams describing reactions. Relations between three-meson vertices involving different members of the same I-spin and U-spin multiplets are easily obtained by coupling I spins and U spins in the standard manner described above with additional simplifications introduced by the Bose statistics of the mesons and the requirement of invariance under charge conjugation.

Let us consider the decay of a neutral vector meson into two charged pseudoscalar mesons. The K^{*0}, the $\overline{K^{*0}}$, and the linear combination $\{\frac{1}{2}\rho^0 + \frac{1}{2}\sqrt{3}\phi^{(0)}\}$ form a U-spin triplet with U = 1. The charged π and K mesons form two U-spin doublets. We thus obtain

$$\langle\{\tfrac{1}{2}\rho^0 + \tfrac{1}{2}\sqrt{3}\,\phi^{(0)}\}\,|\,K^+K^-\rangle : \langle\{\tfrac{1}{2}\rho^0 + \tfrac{1}{2}\sqrt{3}\,\phi^{(0)}\}\,|\,\pi^+\pi^-\rangle : \langle K^{*0}\,|\,K^+\pi^-\rangle$$

$$= \left(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2}\ \middle|\ 1\ 0\right) : \left(\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2}\ \frac{1}{2}\ \middle|\ 1\ 0\right) : \left(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \middle|\ 1\ 1\right). \tag{9.1}$$

We note that since the two pions are spinless bosons in the same I-spin multiplet, they can be in an antisymmetric spatial state only if they are antisymmetric in I-spin, i.e., only if they have T = 1. To make a vector meson with odd parity, the two pions must be in an antisymmetric state. They therefore have T = 1. A T = 0 vector meson therefore cannot decay into two pions. Thus

$$\langle\phi^{(0)}\,|\,\pi^+\pi^-\rangle = 0. \tag{9.2}$$

This result can also be obtained directly from conservation of G parity. Discarding the term describing the decay of the ϕ into two pions leaves

$$\langle K^{*0}\,|\,K^+\pi^-\rangle = \frac{1}{\sqrt{2}}\,\langle\rho^0\,|\,\pi^+\pi^-\rangle\,, \tag{9.3a}$$

$$\langle\tfrac{1}{2}\rho^0 + \frac{\sqrt{3}}{2}\,\phi^{(0)}\,|\,K^+K^-\rangle = -\tfrac{1}{2}\,\langle\rho^0\,|\,\pi^+\pi^-\rangle\,. \tag{9.3b}$$

The first of these relations allows us to predict the ratio of the width of the K^{0*} to the width of the ρ. To do this we need to introduce the neutral decay mode of the K^{0*} which is related to the charged mode by I-spin coupling. This relation is

$$\frac{\langle K^{*0}\,|\,K^0\pi^0\rangle}{\langle K^{*0}\,|\,K^+\pi^-\rangle} = \frac{(\frac{1}{2}\ 1\ -\frac{1}{2}\ 0\,|\,\frac{1}{2}\ \ \frac{1}{2})}{(\frac{1}{2}\ 1\ \frac{1}{2}\ -1\,|\,\frac{1}{2}\ \ \frac{1}{2})} = -\frac{1}{\sqrt{2}}\ . \tag{9.4a}$$

Thus

$$\langle K^{*0}\,|\,K^0\pi^0\rangle = -\tfrac{1}{2}\,\langle\rho^0\,|\,\pi^+\pi^-\rangle \tag{9.4b}$$

and

$$\frac{|\langle K^{*0}|K^+\pi^-\rangle|^2 + |\langle K^{*0}|K^0\pi^0\rangle|^2}{|\langle\rho^0|\pi^+\pi^-\rangle|^2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}\ . \tag{9.4c}$$

To obtain the ratio of the K^* and ρ widths, the factor 3/4 must be multiplied by phase-space factors to include the effect of the different momenta of the two final states. The result is in reasonable agreement with the experimental value of this ratio which is about 1/2.

Let us now consider the decay of neutral vector mesons into K^0 and $\overline{K^0}$. The neutral K mesons are members of the same U = 1 multiplet and must again be in the space-antisymmetric p state. By an argument directly analogous to the I-spin argument employed above for the two-pion system, we see that the $K^0\overline{K^0}$ system in an antisymmetric p state must also be antisymmetric in U spin and must have U = 1. Thus only the U = 1 linear combination

$$\tfrac{1}{2}\rho^0 + \tfrac{1}{2}\sqrt{3}\,\phi^{(0)}$$

is coupled to the $K^0\overline{K^0}$ system and we obtain

$$\langle\rho^0|K^0\overline{K^0}\rangle = \frac{1}{\sqrt{3}}\langle\phi^{(0)}|K^0\overline{K^0}\rangle\,. \tag{9.5}$$

This result can also be expressed in terms of the charged K mode as

$$\langle\rho^0|K^+K^-\rangle = -\frac{1}{\sqrt{3}}\langle\phi^{(0)}|K^+K^-\rangle\,, \tag{9.6}$$

where the negative sign comes from I-spin coupling of $\frac{1}{2} + \frac{1}{2}$ to 1 on the left-hand side and to zero on the right. Then by substituting Eq. (9.6) into Eq. (9.3b) we can relate the ϕ decay to the ρ decay through the expression

$$\langle \phi^{(0)} | K^+K^- \rangle = -\frac{\sqrt{3}}{2} \langle \rho^0 | \pi^+\pi^- \rangle . \tag{9.7}$$

These results expressed in terms of the $\phi^{(0)}$ are easily expressed in terms of the physical ϕ and ω vector mesons by noting that only the $\phi^{(0)}$ linear combination of the physical ϕ and ω can contribute to this decay. The unitary singlet has U = 0 and is not coupled to the $K^0\overline{K^0}$ system which has U = 1. Thus

$$\langle \phi | K^+K^- \rangle = -\tfrac{1}{2} \sqrt{3} \cos \lambda \langle \rho^0 | \pi^+\pi^- \rangle , \tag{9.8a}$$

$$\langle \rho^0 | K^+K^- \rangle : \langle \phi | K^+K^- \rangle : \langle \omega | K^+K^- \rangle = 1 : -\sqrt{3} \cos \lambda : \sqrt{3} \sin \lambda . \tag{9.8b}$$

Equation (9.8a) relates the width of the ϕ to the width of the ρ. Equation (9.8b) can be interpreted as giving the ratio of the production matrix elements for ρ^0, ϕ, and ω production by a $K\overline{K}$ vertex. For example, in K^-p reactions which go via one-K exchange, the ratio is

$$\langle K^-p | \rho^0\Lambda \rangle : \langle K^-p | \omega\Lambda \rangle : \langle K^-p | \phi\Lambda \rangle = 1 : \sqrt{3} \sin \lambda : -\sqrt{3} \cos \lambda . \tag{9.9}$$

10. THE MASS FORMULA

An outstanding success of unitary symmetry theory has been the prediction of the splittings of the masses within an SU_3 multiplet. If strong interactions were exactly invariant under SU_3, all members of the same U-spin multiplet would have the same mass. To obtain a mass splitting that can be predicted by theory, it is necessary to assume that the SU_3 symmetry is broken by some kind of interaction that, although not invariant under SU_3, has simple transformation properties. Furthermore, since I spin is known to be conserved to a very good approximation in strong interactions, the symmetry-breaking part of the strong interaction should still conserve I-spin.

Let us now examine the possible transformation properties of such an interaction under U spin. It cannot be a U-spin scalar; if it conserved both U spin and I spin it would not break the symmetry. The simplest possibility is that it should be some linear combination of a scalar and a vector under U spin as indicated by the interaction M in Table I. The mass formula is obtained by assuming that mass splittings are obtained by taking the expectation value of an operator which has these transformation properties.*

The values of the mass splittings predicted by this theory are easily obtained by use of U spin as follows.

a. The masses of the $\frac{3}{2}^+$ baryon resonances.

Consider the U-spin quartet of negatively charged baryons: Ω^-, Ξ^{*-}, Y_1^{*-}, and N^{*-}. These have $U = \frac{3}{2}$ and $U_z = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}$, and $+\frac{3}{2}$, respectively. Let us consider the expectation value of the mass-splitting operator in this U-spin multiplet. The U-spin scalar part of the operator has the same expectation value for all states in the same U-spin multiplet and therefore does not give any mass splitting. The U-spin vector part gives a splitting which is proportional to U_z within the same U-spin multiplet. Thus the mass splitting of the four members of the quartet is proportional to U_z and the mass spacings are all equal; i.e., there are four equally spaced energy levels.

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* This seems to be assuming that the mass splittings can be treated by first-order degenerate perturbation theory. This is clearly unjustified because the splittings are not small. However, the results agree with experiment so well that the approach seems to be valid, even though it cannot be justified on the basis of first-order perturbation theory. Such situations have arisen before in physics — for example, in the nuclear shell model. One might expect that the treatment, which formally resembles first-order perturbation theory, actually includes many higher order effects as well.

b. The mass splitting in the baryon octet.

Consider the neutral members of the U-spin triplet $[\Xi^0, (\Sigma^0 + \sqrt{3}\,\Lambda)/2$, and n], which are states with U = -1 and U_z = 1, 0, and +1, respectively. Here again the expectation value of the U-spin scalar part of the mass-splitting operator is the same for all three states. Let us denote this by S. The expectation value of the U-spin vector part of the mass-splitting operator is proportional to U_z. If the proportionality factor is denoted by V, the expectation value is therefore -V, 0, and +V, respectively, for the three states. We can therefore write

$$\langle \Xi^0 | M | \Xi^0 \rangle = \langle S \rangle - \langle V \rangle , \tag{10.1a}$$

$$\langle -\tfrac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda | M | -\tfrac{1}{2}\Sigma^0 + \frac{\sqrt{3}}{2}\Lambda \rangle = \langle S \rangle , \tag{10.1b}$$

$$\langle n | M | n \rangle = \langle S \rangle + \langle V \rangle , \tag{10.1c}$$

where M represents the mass-splitting operator. Since M conserves I spin, it has no off-diagonal elements between the Λ and the Σ^0. Equation (10.1b) can therefore be rewritten

$$\frac{1}{4}\langle \Sigma^0 | M | \Sigma^0 \rangle + \frac{3}{4}\langle \Lambda | M | \Lambda \rangle = \langle S \rangle . \tag{10.1d}$$

Combining these equations leads to the result

$$\frac{\langle n | M | n \rangle + \langle \Xi^0 | M | \Xi^0 \rangle}{2} = \frac{\langle \Sigma^0 | M | \Sigma^0 \rangle + 3\langle \Lambda | M | \Lambda \rangle}{4} . \tag{10.2}$$

11. STATIC ELECTROMAGNETIC PROPERTIES

Any electromagnetic operator E is a U-spin scalar. Expectation values of such an operator in states of the same U-spin multiplet are therefore all equal.

Consider for example the baryon octet. The proton and the Σ^+ are members of the same U-spin doublet, and similarly for the Σ^- and Ξ^-. The neutron and Ξ^0 are members of the same U-spin triplet. Therefore for any electromagnetic operator E we have

$$\langle p|E|p\rangle = \langle \Sigma^+|E|\Sigma^+\rangle , \tag{11.1a}$$

$$\langle \Sigma^-|E|\Sigma^-\rangle = \langle \Xi^-|E|\Xi^-\rangle , \tag{11.1b}$$

$$\langle n|E|n\rangle = \langle \Xi^0|E|\Xi^0\rangle . \tag{11.1c}$$

If E represents the magnetic-moment operator, then Eqs. (11.1) predict that the magnetic moments of the proton and Σ^+ are equal, and similarly for the Σ^- and Ξ^- and for the neutron and the Ξ^0. On the other hand if E is interpreted to be the electromagnetic mass-splitting operator, Eqs. (11.1) can be combined to obtain a relation between the mass splittings in the three isotopic multiplets, namely

$$m_p - m_n = m_{\Sigma^+} - m_{\Sigma^-} + m_{\Xi^-} - m_{\Xi^0} . \tag{11.2}$$

12. MESON-BARYON RESONANCE COUPLINGS

The coupling of an excited baryon resonance to a baryon and a pseudoscalar meson is of interest both for the consideration of the decay of the baryon resonance and also as parts of diagrams describing reactions.

Consider the coupling of the negatively-charged baryon resonances (N^{*-}, Y^{*-}, Ξ^{*-}, and $\underline{\Omega^-}$) to a negative baryon (Σ^- or Ξ^-) and a neutral meson (K^0, π^0, η, or $\overline{K^0}$). The negative baryon resonances form a U-spin quartet ($U = \frac{3}{2}$). The negative baryons form a U-spin doublet ($U = \frac{1}{2}$). The neutral mesons form a triplet with U = 1 and a singlet with U = 0. There can be no contribution from the U = 0 state since U spins of 0 and 1/2 cannot be coupled to a total U = 3/2. Only the particular linear combination of π^0 and η which has U = 1 can contribute to this coupling. We can therefore write the transition matrix elements for all of these couplings in terms of a single amplitude $\underline{a}$ with coefficients involving the coupling of a spin $\frac{1}{2}$ and a spin 1 to obtain a spin $\frac{3}{2}$. These matrix elements are

$$\langle N^{*-} | \Sigma^- K^0 \rangle = \left(\tfrac{1}{2} \; 1 \; \tfrac{1}{2} \; 1 \middle| \tfrac{3}{2} \; \tfrac{3}{2} \right) a = a , \tag{12.1a}$$

$$\langle Y^{*-} | \Xi^- K^0 \rangle = \left(\tfrac{1}{2} \; 1 \; -\tfrac{1}{2} \; 1 \middle| \tfrac{3}{2} \; \tfrac{1}{2} \right) a = \sqrt{\tfrac{1}{3}} \; a , \tag{12.1b}$$

$$\langle Y^{*-} | \Sigma^- \pi^- \rangle = \tfrac{1}{2} \left(\tfrac{1}{2} \; 1 \; \tfrac{1}{2} \; 0 \middle| \tfrac{3}{2} \; \tfrac{1}{2} \right) a = \sqrt{\tfrac{1}{6}} \; a , \tag{12.1c}$$

$$\langle Y^{*-} | \Sigma^- \eta^- \rangle = \tfrac{\sqrt{3}}{2} \left(\tfrac{1}{2} \; 1 \; \tfrac{1}{2} \; 0 \middle| \tfrac{3}{2} \; \tfrac{1}{2} \right) a = \sqrt{\tfrac{1}{2}} \; a, \tag{12.1d}$$

$$\langle \Xi^{*-} | \Sigma^- \overline{K^0} \rangle = \left(\tfrac{1}{2} \; 1 \; \tfrac{1}{2} \; -1 \middle| \tfrac{3}{2} \; -\tfrac{1}{2} \right) a = \sqrt{\tfrac{1}{3}} \; a, \tag{12.1e}$$

$$\langle \Xi^{*-} | \Xi^- \pi^0 \rangle = \tfrac{1}{2} \left(\tfrac{1}{2} \; 1 \; -\tfrac{1}{2} \; 0 \middle| \tfrac{3}{2} \; -\tfrac{1}{2} \right) a = \sqrt{\tfrac{1}{6}} \; a , \tag{12.1f}$$

$$\langle \Xi^{*-} | \Xi^- \eta \rangle = \tfrac{\sqrt{3}}{2} \left(\tfrac{1}{2} \; 1 \; -\tfrac{1}{2} \; 0 \middle| \tfrac{3}{2} \; -\tfrac{1}{2} \right) a = \sqrt{\tfrac{1}{2}} \; a , \tag{12.1g}$$

$$\langle \Omega^- | \Xi^- \overline{K^0} \rangle = \left(\tfrac{1}{2} \; 1 \; -\tfrac{1}{2} \; -1 \middle| \tfrac{3}{2} \; -\tfrac{3}{2} \right) a = a. \tag{12.1h}$$

The coupling of the negative baryon resonances to a neutral baryon and a negative meson can be analyzed similarly. Again there is a single amplitude which we can call b, and the Clebsch-Gordan coefficients involve the coupling of a spin $\frac{1}{2}$ and a spin 1 to get a total spin of $\frac{3}{2}$. However, in this case it is the neutral baryon which has U-spin 1 and the negative meson which U-spin $\frac{1}{2}$. The matrix elements for these couplings are

$$\langle N^{*-} | \pi^- n\rangle = \left(\tfrac{1}{2}\ 1\ \tfrac{1}{2}\ 1 \middle| \tfrac{3}{2}\ \tfrac{3}{2}\right) b = b\,, \tag{12.2a}$$

$$\langle Y^{*-} | K^- n\rangle = \left(\tfrac{1}{2}\ 1\ -\tfrac{1}{2}\ 1 \middle| \tfrac{3}{2}\ \tfrac{1}{2}\right) b = \sqrt{\tfrac{1}{3}}\, b, \tag{12.2b}$$

$$\langle Y^{*-} | \pi^- \Sigma^0\rangle = \tfrac{1}{2}\left(\tfrac{1}{2}\ 1\ \tfrac{1}{2}\ 0 \middle| \tfrac{3}{2}\ \tfrac{1}{2}\right) b = \sqrt{\tfrac{1}{6}}\, b\,, \tag{12.2c}$$

$$\langle Y^{*-} | \pi^- \Lambda\rangle = \tfrac{\sqrt{3}}{2}\left(\tfrac{1}{2}\ 1\ \tfrac{1}{2}\ 0 \middle| \tfrac{3}{2}\ \tfrac{1}{2}\right) b = \sqrt{\tfrac{1}{3}}\, b\,, \tag{12.2d}$$

$$\langle \Xi^{*-} | \pi^- \Xi^0\rangle = \left(\tfrac{1}{2}\ 1\ \tfrac{1}{2}\ -1 \middle| \tfrac{3}{2}\ -\tfrac{1}{2}\right) b = \sqrt{\tfrac{1}{3}}\, b\,, \tag{12.2e}$$

$$\langle \Xi^{*-} | K^- \Sigma^0\rangle = \tfrac{1}{2}\left(\tfrac{1}{2}\ 1\ -\tfrac{1}{2}\ 0 \middle| \tfrac{3}{2}\ -\tfrac{1}{2}\right) b = \sqrt{\tfrac{1}{6}}\, b\,, \tag{12.2f}$$

$$\langle \Xi^{*-} | K^- \Lambda\rangle = \tfrac{\sqrt{3}}{2}\left(\tfrac{1}{2}\ 1\ -\tfrac{1}{2}\ 0 \middle| \tfrac{3}{2}\ -\tfrac{1}{2}\right) b = \sqrt{\tfrac{1}{2}}\, b\,, \tag{12.2g}$$

$$\langle \Omega^- | K^- \Xi^0\rangle = \left(\tfrac{1}{2}\ 1\ -\tfrac{1}{2}\ -1 \middle| \tfrac{3}{2}\ -\tfrac{3}{2}\right) b = b\,. \tag{12.2h}$$

The amplitudes a and b are not independent but can be related by I-spin considerations. The simplest way to obtain this relation is from the Ω^- which has T = 0 and is coupled to the opposite members of two I-spin doublets, namely $|K^-\Xi^0\rangle$ and $|\overline{K^0}\Xi^-\rangle$. Since the linear combination $|K^-\Xi^0\rangle - |\overline{K^0}\Xi^-\rangle$ has T = 0, we obtain

$$a = -b \,. \tag{12.3}$$

The same result is obtained from the $\langle Y^{*-} | \Sigma^- \pi^0 \rangle$ and $\langle Y^{*-} | \Sigma^0 \pi^- \rangle$ amplitudes which go through a single I-spin channel with T = 1.

The matrix elements for the other charge states of the baryon resonances are obtained from the above results by simple I-spin coupling.

ACKNOWLEDGEMENT

An extensive bibliography on unitary symmetry would be inconsistent with the informal nature of this report and would be out of date before the multilith masters could be run through the machine. An incomplete bibliography is a good way to make enemies. Therefore no references are given in this report to papers on unitary symmetry. The reader may prepare his own by picking a recent issue of Physical Review Letters or Physics Letters and working backwards. The author gratefully acknowledges stimulating discussions with an enormous stack of papers, letters, and preprints, which have aided in the preparation of this seminar lecture.

APPENDIX

Determination of the Mixing Parameter for the Center of the Octet Diagrams

The Λ and the Σ^0, which occupy the same point on the diagram (fig. 1a), are I-spin eigenstates but are not U-spin eigenstates. The particular mixtures of Λ and Σ^0 which are U-spin eigenstates can be determined from standard angular-momentum algebra and the additional commutation requirement

$$[\,U_-, \tau_+\,] = 0\,. \tag{A1}$$

The vanishing of the commutator (A1) follows from SU_3 group theory, and can also be understood simply as follows. We wish the set of I-spin, U-spin, and V-spin operators to form a closed set analogous to angular-momentum operators; i.e., the commutator of any two operators should give a linear combination of the operators of the set. (In group-theoretical language, this is the requirement for a "Lie algebra.") The commutator of U_- and τ_+ cannot be expressed in this way; U_1 increases T_z by 1/2 and τ_+ increases T_z by 1. Thus the products $U_-\tau_+$ and τ_+U_- both increase T_z by 3/2 and the commutator must also have this property. Since there is no operator in the set which can change T_z by 3/2, the commutator (A1) must vanish if the set of operators is to be a closed set (Lie algebra) like angular momenta.

We shall now determine the particular linear combination of Λ and Σ^0 states that belongs in the U-spin triplet (U = 1). Let us call this particular linear combination

$$|U = 1,\ U_z = 0\rangle = \alpha|\Sigma^0\rangle + \beta|\Lambda\rangle \tag{A2}$$

where α and β are constants to be determined. Since the neutron is the member of the same U-spin triplet with $U_z = +1$, the standard lowering operator relation analogous to angular momentum gives

$$U_-|n\rangle = \sqrt{2}\,\{\alpha|\Sigma^0\rangle + \beta|\Lambda\rangle\}\ . \tag{A3}$$

Operating on Eq. (A3) with the I-spin raising operator τ_+ then gives

$$\tau_+U_-|n\rangle = 2\alpha|\Sigma^+\rangle\ . \tag{A4}$$

The operation of these U-spin and I-spin operators on the neutron state is easily calculated in the reverse order in which the intermediate state is the proton state and there are no ambiguities. Since the operators τ_+ and U_- commute, it follows that

$$\tau_+U_-|n\rangle = U_-\tau_+|n\rangle = U_-|p\rangle = |\Sigma^+\rangle\ . \tag{A5}$$

Then from Eqs. (A4) and (A5),

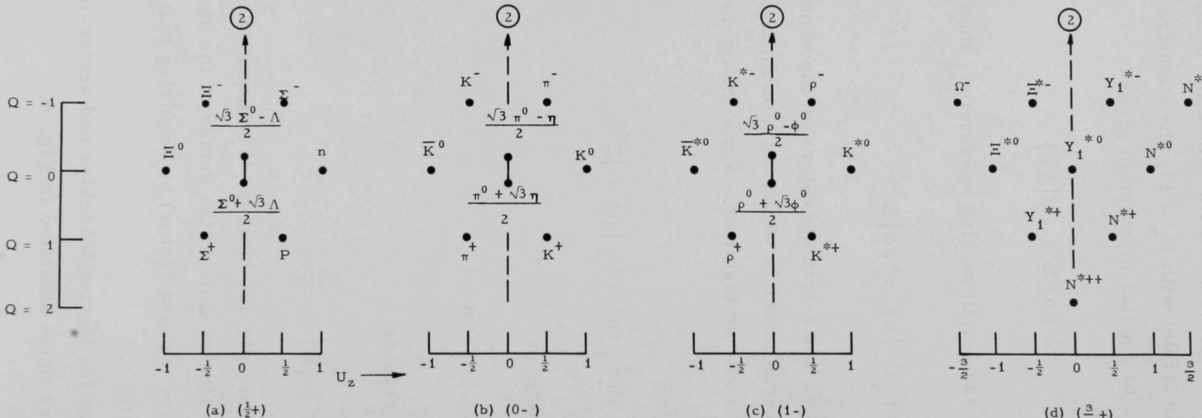
$$\alpha = \tfrac{1}{2}\ . \tag{A6}$$

Hence normalization of (A3) yields

$$|\beta| = \frac{\sqrt{3}}{2}\ . \tag{A7}$$

Since the phase of β is not uniquely determined, it is fixed by convention to be positive. The U-spin singlet state at the center of the octet diagram is then determined (except for a phase factor) by requiring that it be orthogonal to the triplet state.

The treatment for the pseudoscalar and vector-meson octets is identical. The results are those exhibited in Fig. 4.



$$\phi^{(0)} = (\cos \lambda)\phi - (\sin \lambda)\,\omega$$

$$\omega^{(0)} = (\sin \lambda)\phi + (\cos \lambda)\omega$$

Fig. 4. Q vs. U_z Plots for Several SU_3 Multiplets.